

Geometric Structure

Activity: Mad as a Hatter

TEKS: (G.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system.

The student is expected to:

(A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning and theorems;

(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.

The student is expected to:

(A) determine the validity of a conditional statement, its converse, inverse and contrapositive;

(C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false;

(G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal or symbolic) in order to solve problems.

Overview: In this activity, students will learn and use new vocabulary: conditional statement, converse, inverse, contrapositive, counterexample, biconditional statement, and negation. They will then practice their proficiency through writing conditional statements.

Materials: *Mad as a Hatter* handout
Easel paper and markers (not required, but may be used for presenting to the class)

Grouping: pairs or groups of 4

Lesson:

Procedures	Notes
1. Introduce the new vocabulary: conditional statement, converse, inverse, contrapositive, counterexample, biconditional statement, and negation. Then use examples to illustrate them.	Example: Mr. French is the only calculus teacher. Mr. French, Ms. Anderson, Ms. Allen and Ms. Short teach pre-calculus. The conditional statement is "If I take calculus, then Mr. French is my teacher." Have the students write the converse, inverse, and contrapositive and identify as true or false.

Procedures	Notes
2. Divide students into groups and distribute the activity pages. Direct students to work through the activities in their groups.	Circulate among the groups to clarify the terms if needed and to answer questions.
3. Bring the class back together to discuss the statements and clarify any remaining questions. Also have students check their work orally.	The teacher may wish to let selected groups present specified problems on easel paper. All solutions are given including one for the opening example of Mr. French.

Extensions: Assign project that has students write their own conditional statement book. The teacher may want to read to the class the book, *If You Give a Mouse a Cookie* by Laura Numeroff, ISBN 0694006300, as an example of a story based upon connected conditional statements. A sample rubric, *Conditional Statement Book Project*, is provided to use if this project is assigned to students.

Resources: Lesson was adapted from Dana Center Geometry Assessments and the Teacher Quality Grant developed by Rice University

Mad as a Hatter

"Then you should say what you mean, " the March Hare went on. "I do," Alice hastily replied, "at least I mean what I say—that's the same thing, you know." "Not the same thing a bit!" said the Hatter. "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see!'"

Lewis Carroll, "Alice in Wonderland"

You will need to be able to use the vocabulary to do this activity. Complete the blanks to check your memory.

A _____ statement can be written in the form "if P, then Q." This "if, then" statement can be _____ or _____. The "if" part is called the _____, and the "then" part is called the _____. To write the _____ of a conditional statement, interchange the hypothesis and the conclusion. To write the _____ of a conditional statement, negate the hypothesis and conclusion. To write the _____ of a conditional statement, interchange the hypothesis and conclusion of the inverse.

For each of the following conditional statements:

- Determine if the original statement is true or false. Rewrite it as a conditional statement if it is not already in that form.
- Explain in writing the reasoning for each of your choices.
- Write the converse, inverse, and contrapositive.
- Determine if the converse, inverse and contrapositive statements are true or false.
- Explain in writing the reasoning for each of your choices.

1. If a number is divisible by 4, then it is divisible by 2.

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

Inverse: _____

This statement is: ____True ____False. Explain your reasoning.

Contrapositive: _____

This statement is: ____True ____False. Explain your reasoning.

2. If it is raining in Las Vegas, then it is sunny in Nevada.

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

Inverse: _____

This statement is: ____True ____False. Explain your reasoning.

Contrapositive: _____

This statement is: ____True ____False. Explain your reasoning.

3. A rhombus is a quadrilateral with four congruent sides.

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

Inverse: _____

This statement is: ____True ____False. Explain your reasoning.

Contrapositive: _____

This statement is: ____True ____False. Explain your reasoning.

4. The sum of the measures of the interior angles of a triangle is 180.

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

Inverse: _____

This statement is: ____True ____False. Explain your reasoning.

Contrapositive: _____

This statement is: ____True ____False. Explain your reasoning.

5. Write a real-world example of a conditional statement with a true converse.

Conditional Statement: _____

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

6. Write a real-world example of a conditional statement with a false converse.

Conditional Statement: _____

This statement is: ____True ____False. Explain your reasoning.

Converse: _____

This statement is: ____True ____False. Explain your reasoning.

7. What conclusions can be made about the truth of converse, inverse, and contrapositive statements when the conditional statement is true?

ANSWER KEY FOR MAD AS A HATTER

EXAMPLE:

Conditional Statement: If I take calculus, then Mr. French is my teacher.

Converse: If Mr. French is my teacher, then I take calculus.

False, because I could take pre-calculus from Mr. French.

Inverse: If I do not take calculus, then Mr. French is not my teacher.

False, because I could take pre-calculus from Mr. French.

Contrapositive: If Mr. French is not my teacher, then I don't take calculus.

True

A **conditional** statement can be written in the form "if P, then Q." This "if, then" statement can be **true** or **false**. The "if" part is called the **hypothesis** and the "**then**" part is called the conclusion. To write the **converse** of a conditional statement, interchange the hypothesis and the conclusion. To write the **inverse** of a conditional statement, negate the hypothesis and conclusion. To write the **contrapositive** of a conditional statement, interchange the hypothesis and conclusion of the inverse.

1. Conditional Statement: True. Since 4 is divisible by 2, then numbers divisible by 4 must also be divisible by 2.

Converse: If a number is divisible by 2, then it is divisible by 4. False. Numbers such as 2, 6, and 10 are divisible by 2, but not by 4.

Inverse: If a number is not divisible by 4, then it is not divisible by 2. False. Numbers such as 6 and 10 are divisible by 2, but not by 4.

Contrapositive: If a number is not divisible by 2, then it is not divisible by 4. True. Since 4 is divisible by 2, then numbers divisible by 4 must also be divisible by 2.

2. Conditional Statement: False. If it is raining in Las Vegas, then it is sunny throughout the entire state of Nevada.

Converse: If it is sunny in Nevada, then it is raining in Las Vegas. False. Las Vegas would be sunny if it were sunny in Nevada; therefore, it can't be raining in Las Vegas.

Inverse: If it is not raining in Las Vegas, then it is not sunny in Nevada. False. If it is not raining in Las Vegas, then it is sunny in Las Vegas and Las Vegas is in Nevada.

Contrapositive: If it is not sunny in Nevada, then it is not raining in Las Vegas. False. If it is not sunny in the entire state of Nevada, then it is raining in the entire state.

3. Conditional Statement: If a quadrilateral is a rhombus, then it has four congruent sides. True. This is the definition of a rhombus.

Converse: If a quadrilateral has four congruent sides, then it is a rhombus. True. This is the definition of a rhombus.

Inverse: If a quadrilateral is not a rhombus, then it does not have four congruent sides. True. This is true by the definition of a rhombus.

Contrapositive: If a quadrilateral does not have four congruent sides, then it is not a rhombus. True. This is true by the definition of a rhombus.

4. Conditional Statement: If a polygon is a triangle, then the sum of the measures of the interior angles is 180 degrees. True.

Converse: If the sum of the measures of the interior angles is 180 degrees, then the polygon is a triangle. True.

Inverse: If a polygon is not a triangle, then the sum of the measures of the interior angles is not 180 degrees. True.

Contrapositive: If the sum of the measures of the interior angles is not 180 degrees, then the polygon is not a triangle. True.

5. Possible answer: My cat and dog always eat together.

Conditional Statement: If my cat eats, then my dog eats.

Converse: If my dog eats, then my cat eats.

6. Possible answer: Every Mathlete at Lanier Middle School is an 8th grade student.

Conditional Statement: If a Lanier Middle School student is a Mathlete, then he/she is an 8th grade student.

Converse: If a Lanier Middle School student is an 8th grade student, then he/she is a Mathlete. False. We do not know that every 8th grade student is a Mathlete.

7. When two statements are both true or both false, they form a biconditional statement. A conditional and its contrapositive form a biconditional statement. The converse and inverse statements of a conditional statement also form a biconditional statement.

CONDITIONAL STATEMENT BOOK PROJECT

Create a conditional statement book of your own. It can be on any topic as long as it is school appropriate. It must contain eight conditional statements and the converse of two of these statements. Extra credit will be given for the inverse or contrapositive of a conditional statement. You will evaluate your own work by completing this rubric and returning it with your project. Books will not be accepted without this accompanying page.

_____ (24 points) Three points for each conditional statement used. You must list the statements and identify the hypothesis and conclusion of each.

_____ (10 points) Five points for each converse statement used. You must list the converse statements.

_____ (6 points) Creativity and Originality

_____ (10 points) Effort

_____ (5 points) Extra Credit for an inverse or contrapositive statement.

_____ Total